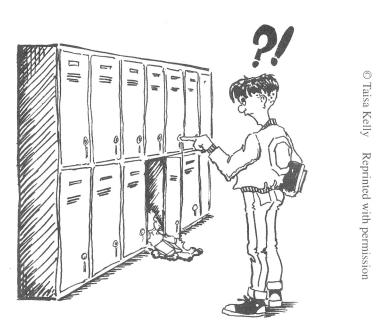## The Painted Lockers

The lockers in Cretin High School are numbered 1 through 1000. Cretin's school colors are a fashionable mauve and vermilion. All of the lockers are initially painted mauve. After an excessively enthusiastic pep rally, Cretin's 1000 students go on a painting rampage. The first student runs down the halls and paints all of the lockers vermilion. The second student runs down the halls and repaints all of the even-numbered lockers mauve. The third student runs down the halls and repaints lockers 3, 6, 9, ..., 999 in the opposite color that he finds them. The fourth student runs down the halls and does the same with those lockers whose numbers are multiples of 4. This goes on and on and on, until finally (late that night), the thousandth student repaints locker 1000. After that peppy day, which lockers are painted mauve, and which lockers are painted vermilion? (Hint: imagine that the school only has 10 lockers, and work out what happens in that case. Then try it with 20 lockers. Then make a conjecture for the general case. Finally, try to prove it.)

The answer is in *Solution to the Painted Lockers Problem* (p. 134).