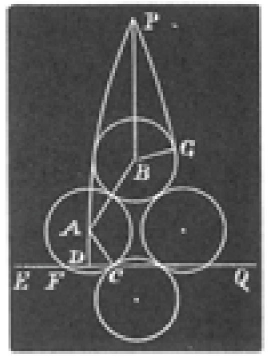
Solution by WALTER SIVERLY, Oil City, Venango Co.; and the PROPOSER.

The centers of the circumscribing spheres are on the surface of a sphere 13 inches in diameter. Let the extremities of any diameter of this sphere be taken as poles, and the great circle midway between them be called the equator. The distance between the centers of two consecutive circumscribing spheres is $2\sin^{-1}(\frac{1}{13}) = 8° 49' 24''$.



Let P be a pole, EQ a portion of the equator, and let a row of 40 spheres be arranged each side of the equator as in the figure. The portion of the equator intercepted by each sphere, as FC, is $360° \div 80 = 4° 30'$.

In the right-angled spherical triangle ACD, $AC = 4° 24' 42''$, $CD = 2° 15'$, to find $AD = 3° 47' 45''$. In the spherical triangle PAB, $PA = 90° - 3° 47' 45'' = 86° 12' 15''$, $AB = 8° 49' 24''$, $APB = 4° 30'$, to find $PB = 78° 35' 54''$. In the right-angled spherical triangle PBG, $PB = 78° 35' 54''$, $BG = 4° 24' 42''$, to find $BPG = 4° 29' 22''$. Hence the second row will contain 40 spheres.

The succeeding rows will be irregular, each fitting only partially between the spheres of the preceding one. Proceeding in a similar manner for the polar distance of one sphere in each row, and the corresponding number of spheres, the following result is obtained:

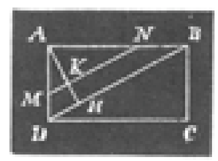
2 rows of	40 spheres each,	polar distance	86° 12' 15'',	give	80 spheres;
2 "	" 40	" " " "	78° 35' 54''	"	80 "
2 "	" 38	" " " "	69° 51'	"	76 "
2 "	" 36	" " " "	61° 41'	"	72 "
2 "	" 32	" " " "	53° 07'	"	64 "
2 "	" 28	" " " "	44° 38'	"	56 "
2 "	" 24	" " " "	36° 03'	"	48 "
2 "	" 19	" " " "	27° 25'	"	38 "
2 "	" 13	" " " "	19° 48'	"	26 "
2 "	" 7	" " " "	11° 00'	"	14 "
2 "	" 1	" " " "	0° 00'	"	2 "
Adding, whole number				=	556 spheres.

The polar distances of other spheres in the rows are slightly different, but the difference is not sufficient to change the result.

190 **190.**—Proposed by F. P. MATZ, M. A., late Professor of Mathematics, Military and Scientific School, King's Mountain, North Carolina.

A given rectangle is divided at random by a line parallel to one of its diagonals, and then two points are taken at random within the rectangle; find the chance that both points are on the same side of the dividing line.

Solution by ARTEMAS MARTIN, M. A., Member of the London Mathematical Society, Erie, Erie County, Pa.



Put $a = AB = CD$, $b = AD = BC$; then $BD = \sqrt{(a^2+b^2)} = c$, $AH = \frac{ab}{c} = x'$.

Let $x = AK$, then $MN = \frac{c^2x}{ab}$, and area $MAN = \frac{c^2x^2}{2ab} = u$, area $MDCBN = ab - u$.

$$p = \int_0^{x'} [u^2+(ab-u)^2]dx \div \int_0^{x'} a^2b^2dx = \frac{c}{a^3b^3}\int_0^{x'} (a^2b^2 - 2abu + 2u^2)dx$$

$$= \frac{c}{a^3b^3}\int_0^{x'} \left(a^2b^2 - c^2x^2 + \frac{c^4x^4}{2a^2b^2}\right)dx = \frac{23}{30}.$$

This is a remarkable result, being independent of the dimensions of the rectangle.

191 **191.**—Proposed by REUBEN DAVIS, Bradford, Stark County, Illinois.

Find three square numbers such that if to each its root be added, and from each its root be subtracted, the three sums and three remainders shall all be rational squares.

Solution by WILLIAM HOOVER, M. A., Principal of Second District School, Dayton, Ohio.

The problem will be solved if we find three values of x which will satisfy

$$x^2 - x = \square \ldots\ldots (1), \qquad \text{and} \qquad x^2 + x = \square \ldots\ldots (2).$$

Put $x^2 - x = (x-m)^2$, and we get $x = \frac{m^2}{2m-1} \ldots\ldots (3)$. This substituted in (2) gives by reduction,

$$m^2 + 2m - 1 = \square, = (m-t)^2, \text{ say}.$$

This equation gives $m = \frac{t^2+1}{2t+2}$. Let $t = 2$, 5 or 7; then $m = \frac{5}{6}$, $\frac{13}{6}$ or $\frac{25}{8}$; hence $x = \frac{25}{24}$, $\frac{169}{120}$ or $\frac{625}{336}$.

It is plain that the values of x can not be integers, and that an infinite number of answers may be found.