## Introduction

The NYSML and ARML competitions have now become an eagerly awaited annual event, attracting hundreds of enthusiastic students who compete for statewide and national honors. Whereas NYSML (New York State Mathematics League) attracts teams primarily from New York State, ARML (American Regions Mathematics League) draws teams from an ever-widening geographical area.

Both contests are identical in format and offer a variety of challenging problems and activities. Each team is composed of fifteen members who compete jointly and individually in the four major events of the meet. First there is a set of TEAM QUESTIONS, next comes the POWER QUESTION, then a set of INDIVIDUAL QUESTIONS, and finally two RELAY rounds.

The twenty-minute TEAM round consists of ten short questions. Team members usually distribute the problems among themselves, and as students complete their assigned problems, they then help others who are having difficulty. The POWER QUESTION, a challenging problem requiring an in-depth analysis, usually consists of several sections united by a common theme. The team has one hour to produce a mathematically sound, well-written solution, including all requisite proofs. After a short break, all teams gather in a large auditorium for the next two events. In the INDIVIDUAL round, students work independently on the same set of eight questions. These questions are presented in pairs, with ten minutes allotted for each pair. Individual correct answers contribute points to the team score. The results of this round are also used to determine which students will receive special individual awards. In the RELAY round, each team is partitioned into miniteams. The questions are designed so that each question after the first one incorporates the answer from the previous question. Thus, each team member anxiously awaits an answer that is passed back by the person in front of him or her. It is possible for the skillful problem solver to do most of the work before receiving this answer. The speed with which the final question is correctly answered determines the points awarded the team.

This volume contains all the NYSML and ARML questions from 1983 through 1988, along with solutions and an index that classifies the problems according to topic. Although the solutions provided by the authors are not always the shortest or necessarily the most straightforward, they were often chosen for their instructive value. At times we have indicated possible extensions, generalizations, and avenues for further pursuit. Many of these problems, even some of the seemingly humble relay questions, have a future. Indeed, many have been crafted by our creative mathletes into research topics

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and projects for math fairs. It is what happens after the competition that proves to be most important.

The competition itself is not only a day of good mathematical problem solving but also a social event where some of our nation's most talented young people meet, exchange ideas, and renew old friendships. It should be noted, however, that although a competition provides a setting for stimulating mathematical activity, good mathematics should not be practiced only under contest conditions, and a contest should not be an end unto itself. This book provides a large collection of challenging problems that can be used for contest practice or can be the basis for more leisurely study and extensive exploration.

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