21. Expanding, we get $1 + 2ab + a^2b^2 + 1 + 2cd + c^2d^2 + a^2c^2 + b^2d^2 = 1 + (1 + ab + cd)^2 + (ac - bd)^2 \geq 1$.

22. Let M and N denote points of intersection of B_2A_2 with BA_1 and AB_1, respectively (see Figure 6.5). Segment B_2N is a midline in triangle ABB_1, so $B_2N = AB/2$, and similarly, $A_2M = AB/2$. Thus, we have that triangles BMA_2 and ANB_2 are congruent, since they have equal sides $B_2N = MA_2$, equal angles B_2AN and A_2BM, and equal altitudes dropped from vertices A and B, respectively. Consequently, $\angle AB_2A_2 = \angle BA_2B_2$. Using this equality, we can show in the same way that triangles AB_2A_2 and BA_2B_2 are equal; in particular, $\angle BB_2A_2 = \angle AA_2B_2$, and therefore $\angle CA_2B_2 = \angle CB_2A_2$, that is, $\angle CAB = \angle CBA$, since A_2B_2 is parallel to AB.

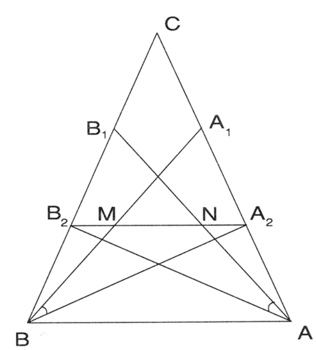

Figure 6.5

23. SOLUTION 1. We will show by induction on the number N of boughs that the total time of the process as well as the number of crows who have flown away does not depend on the order of flights. At the beginning, P crows are sitting on the highest (Nth) bough of the oak. Consider the process on the other boughs of the oak. If this reduced process takes T minutes and the number of flown crows is equal to Q, then we can derive that there must be exactly $P + Q - 1$ crows that have to fly away from the Nth bough (except for the case when $P = Q = 0$; then there would be no crows of this type). Thus, it takes $P + Q - 1$ more minutes, and we can conclude that the whole process would take $T + P + Q - 1$ minutes and that the number of flown crows is equal to $P + Q - 1$ irrespective of the order of flights.