

Grade 8

19. Fifty pawns are placed on a 10×10 board so that 25 of them occupy the lower-left quarter of the board and the other 25 occupy the upper-right quarter. Each pawn can jump over any neighboring one to the next square, if it is free (see Figure 1.3). Is it possible that after several such jumps these pawns will occupy only the left half of the board? [p. 58]

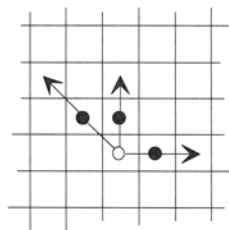


Figure 1.3

20. The coins in a heap are of values 1, 2, 5, 10, 20, and 50 cents, and \$1. Some set of B of these coins has a value of A cents. Prove that some set of A of these coins has a value of B dollars. [p. 58]
21. Given the real numbers $a, b, c,$ and d prove that

$$(1 + ab)^2 + (1 + cd)^2 + (ac)^2 + (bd)^2 \geq 1.$$

[p. 59]

22. A triangle ABC is drawn in the plane. Points A_1, A_2 and B_1, B_2 are taken on sides AC and BC , respectively, so that $CA_1 = A_1A_2 = A_2A$ and $CB_1 = B_1B_2 = B_2B$. Prove that triangle ABC is isosceles if it is known that $\angle A_1BA_2 = \angle B_1AB_2$. [p. 59]
23. Several crows are sitting on the boughs of a big oak. At some moment they begin to change places in the following way. Each minute, a crow's neighbors on the same bough drive her away, and she flies to the next (higher) bough of the oak. If there are no boughs higher than this one, the crow flies away. No two boughs occur at the same height. Prove that the moment when the process will terminate (i.e., when there is no more than one crow on each bough) depends only on the initial arrangement of the crows and not on the order of the flights. [p. 59]