SIAM 80-19. by E. Arthurs and B. W. Stuck

(a) Show that the following inequality holds:

$$\sum_{i=1}^{N} G(S_i, A_i) \geq G\left(\sum_{i=1}^{N} S_i, \sum_{i=1}^{N} A_i\right),$$

where

$$0 \leq A_i < S_i, \qquad S_i = 1, 2, \ldots, \qquad i = 1, \ldots, N,$$

$$G(S, A) = AB(S, A), \qquad B(S, A) = \frac{A^S/S!}{\sum_{k=0}^{S} A^k/k!}.$$

(b) Show that the following inequality holds:

$$\sum_{i=1}^{N} H(S_i, A_i) \geq H\left(\sum_{i=1}^{N} S_i, \sum_{i=1}^{N} A_i\right),$$

where

$$0 \leq A_i < S_i, \qquad S_i = 1, 2, \ldots, \qquad i = 1, \ldots, N,$$

$$H(S, A) = \frac{A}{S - A} C(S, A),$$

$$C(S, A) = \frac{A^S/(S-1)!(S-A)}{\sum_{k=0}^{S-1} A^k/k! + A^S/(S-1)!(S-A)}.$$

AMM E2884. by Lawrence Harris

Let $x_1, \ldots, x_n$ be distinct real numbers. Set

$$S = \sum_{k=1}^{n} (1 + x_k^2)^{n/2}/P(k),$$

where $P(k) = \prod_{j \neq k} |x_k - x_j|$. Prove $S \geq n$. When does $S = n$?

CRUX 992. by Harry D. Ruderman

Let $\alpha = (a_1, a_2, \ldots, a_{mn})$ be a sequence of positive real numbers such that $a_i \leq a_j$ whenever $i < j$, and let $\beta = (b_1, b_2, \ldots, b_{mn})$ be a permutation of α. Prove that

(a) $$\sum_{j=1}^{n} \prod_{i=1}^{m} a_{m(j-1)+i} \geq \sum_{j=1}^{n} \prod_{i=1}^{m} b_{m(j-1)+i}$$

(b) $$\prod_{j=1}^{n} \sum_{i=1}^{m} a_{m(j-1)+i} \leq \prod_{j=1}^{n} \sum_{i=1}^{m} b_{m(j-1)+i}.$$

BALKAN 1984/1.

Let $x_1, x_2, \ldots, x_n$, $(n \geq 2)$ be positive numbers whose sum is 1. Prove that

$$\frac{x_1}{1 + x_2 + x_3 + \cdots + x_n} + \frac{x_2}{1 + x_1 + x_3 + \cdots + x_n} + \cdots + \frac{x_n}{1 + x_1 + x_2 + \cdots + x_{n-1}} \geq \frac{n}{2n-1}.$$

GAZ 68.G. by H.-J. Seiffert

For integers $m \geq 2$, prove or disprove:

$$\sum_{k=1}^{m-1} \frac{1}{2k+1} \leq \ln\sqrt{m} \leq \sum_{k=1}^{m} \frac{1}{2k+1}.$$

MSJ 520. by F. David Hammer

Prove that if $a_1, a_2, \ldots, a_n$ are real numbers such that $a_1 + a_2 + \cdots + a_n = 1$, then

$$a_1^2 + a_2^2 + \cdots + a_n^2 \geq \frac{1}{n}.$$

POLAND 1984/3.

If $a, x_1, x_2, \ldots, x_n$ $(n \geq 2)$ are positive real numbers, prove that

$$\frac{a^{x_1 - x_2}}{x_1 + x_2} + \frac{a^{x_2 - x_3}}{x_2 + x_3} + \cdots + \frac{a^{x_n - x_1}}{x_n + x_1} \geq \frac{n^2}{2(x_1 + x_2 + \cdots + x_n)}$$

and determine when there is equality.

SPECT 15.9. by A. J. Douglas and G. T. Vickers

Let $x_1, x_2, \ldots, x_n$ be real numbers such that $0 \leq x_i \leq 1$ for $i = 1, 2, \ldots, n$. Prove that

$$\frac{1}{n}\sum_{i=1}^{n} x_i^2 - \left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)^2 \leq \begin{cases} \frac{1}{4} & \text{when } n \text{ is even} \\ \frac{1}{4} - \frac{1}{4n^2} & \text{when } n \text{ is odd.} \end{cases}$$

Discuss when equality occurs.

PENT 324. by Michael W. Ecker

Let $x_1, x_2, \ldots, x_n$ be positive numbers whose sum is 1. What is the smallest possible value for the sum of the reciprocals

$$\sum_{i=1}^{n} \frac{1}{x_i}?$$

SSM 3835. by Alan Wayne

Prove that if the sum of n positive numbers is 1, then the sum of their reciprocals is at least n^2.

NAvW 571. by R. J. Stroeker

For any real numbers $x_1, \ldots, x_n$, where $n \geq 2$, define

$$F(x_1, x_2, \ldots, x_n) = \sum_{i=1}^{n} \sum_{j=1}^{n} \left\{ f(x_i - x_j) - f(x_i + x_j) \right\}.$$

Here f is defined by

$$f(x) = \begin{cases} \frac{\sin x}{x} & \text{if } x \neq 0, \\ 1 & \text{if } x = 0. \end{cases}$$

Show that $F(x_1, x_2, \ldots, x_n) \geq 0$. When does equality hold?

Inequalities: fractions

CANADA 1984/5.

Given any seven real numbers, prove that there are two of them, say x and y, such that

$$0 \leq \frac{x - y}{1 + xy} \leq \frac{1}{\sqrt{3}}.$$