Paper folding: regular pentagons

SSM 3661. **by Alan Wayne**

A piece of paper has the shape of a regular pentagon. The paper is folded over once and creased flat so that a vertex of the pentagon coincides with the midpoint of that side which is farthest from the vertex. Show that the length of the crease is one and a half times the length of a side of the pentagon.

Paper folding: regular polygons

CRUX 350. **by W. A. McWorter, Jr.**

What regular n-gons can he constructed by paper folding?

Paper folding: squares

CRUX 292. **by Charles W. Trigg**

Fold a square piece of paper to form four creases that determine angles with tangents of 1, 2, and 3.

Paper folding: strips

MSJ 464.

A strip of adding machine tape is folded making an angle A_0 of 80° with the bottom edge of the tape. Angles A_1, A_2, A_3, ... are formed by successive folds of the edges of the tape to the creases previously obtained (and thereby halving the respective angles). Find the measure of A_{100}.

Parabolas

OSSMB G78.1-3.

Let P, Q, R be three points on a parabola such that their distances from the axis of the parabola are in geometric progression. Show that the tangents to the parabola at P and R meet on the line through Q perpendicular to the axis.

OSSMB G79.1-2.

A chord $y = mx + b$ intersects a parabola $y^2 = 4px$ at $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$. Find the coordinates of P, a point on the parabola, such that $\triangle PP_1P_2$ has maximum area.

NYSMTJ 94. **by H. O. Eberhart**

A nonaxial line passing through the focus of a parabola intersects it in two points, P and Q. Show that

(a) the tangent at P is perpendicular to the tangent at Q;

(b) these tangents intersect on the directrix.

CRUX 445. **by Jordi Dou**

Consider a family of parabolas escribed to a given triangle. To each parabola corresponds a focus F and a point S of intersection of the lines joining the vertices of the triangle to the points of contact with the opposite sides. Prove that all lines FS are concurrent.

Parallelograms

TYCMJ 153. **by K. R. S. Sastry**

Let $c \in (0, 1)$ be given and $A_1A_2A_3A_4$ be a parallelogram of one unit area with $E_i \in A_iA_{i+1}$ such that $A_iE_i/E_iA_{i+1} = c$, $(i = 1, 2, 3, 4;\ A_5 = A_1)$. Set $A_iE_{i+1} \cap A_{i+1}E_{i+2} = P_i$, $(i = 1, 2, 3, 4;\ A_5 = A_1,\ E_5 = E_1,\ E_6 = E_2)$. Determine the area of quadrilateral $P_1P_2P_3P_4$.

NYSMTJ 74. **by Norman Gore**

NYSMTJ OBG3. **by Norman Gore**

In parallelogram $ABCD$, L and M are interior points of sides AD and BC respectively. Let $P = \overline{BL} \cap \overline{AM}$ and $Q = \overline{MD} \cap \overline{CL}$. If the line determined by P and Q is parallel to line AD, show that it bisects $ABCD$.

CRUX 139. **by Dan Pedoe**

Let $ABCD$ be a parallelogram, and suppose a circle γ touches AB and BC and intersects AC in the points E and F. Show that there exists a circle δ which passes through E and F and touches AD and DC.

NYSMTJ OBG1. **by Norman Schaumberger**

Let $ABCD$ be a parallelogram. If a circle passes through A and cuts segments AB, AC, and AD at points P, Q, and R respectively, then prove that

$$AP \times AB + AR \times AD = AQ \times AC.$$

TYCMJ 117. **by Norman Schaumberger**

Let E be the intersection of the diagonals of a parallelogram $ABCD$, and let P and Q be points on a circle with center E. Prove that

$$PA^2 + PB^2 + PC^2 + PD^2 = QA^2 + QB^2 + QC^2 + QD^2.$$

CRUX 322. **by Harry Sitomer**

In parallelogram $ABCD$, $\angle A$ is acute and $AB = 5$. Point E is on AD with $AE = 4$ and $BE = 3$. A line through B, perpendicular to CD, intersects CD at F. If $BF = 5$, find EF.

NYSMTJ 43.

Given perpendicular rays $\overrightarrow{AB}$ and $\overrightarrow{AC}$, let $\overline{PQ}$ be any segment with an endpoint on each ray (other than A). Let X be the point of intersection of the bisectors of the exterior angles at P and Q of $\triangle APQ$. Introduce segments $\overline{XM}$ and $\overline{XN}$ perpendicular to rays $\overrightarrow{AB}$ and $\overrightarrow{AC}$, respectively. Prove that parallelogram $ANXM$ is a square.

SSM 3754. **by Fred A. Miller**

If θ is the angle between the diagonals of a parallelogram whose sides a and b are inclined at an angle α to each other, show that

$$\tan\theta = \frac{2ab\sin\alpha}{a^2 - b^2}.$$

Pentagons

CRUX 232. **submitted by Viktors Linis**

Given are five points A, B, C, D, and E in the plane, together with the segments joining all pairs of distinct points. The areas of the five triangles BCD, EAB, ABC, CDE, and DEA being known, find the area of the pentagon $ABCDE$.

PME 383. **by Norman Schaumberger**

Find a pentagon such that the sum of the squares of its sides is equal to four times its area.

Perspective drawings

CRUX 406. **by W. A. McWorter Jr.**

The figure shows an unfinished perspective drawing of a railroad track with two ties drawn parallel to the line at infinity. Can the remaining ties be drawn, assuming that the actual track has equally spaced ties?